Recovering Classical Gravity from de Sitter Geometry

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April 24, 2025

Abstract

We derive Newtonian gravity as a mean-field effect of a discrete particle model embedded in de Sitter geometry, recovering the familiar $-GM/r^2$ form from the proper acceleration profile of de Sitter space. We then propose a quantum extension of this geometric particle, modeled as a bound state in a harmonic oscillator potential derived from the same background acceleration field.

1 Introduction

The quest to understand gravity from both classical and quantum perspectives has motivated a variety of approaches that reinterpret gravitational dynamics in terms of geometric principles. Among these, de Sitter space—a solution to Einstein's equations with a positive cosmological constant—provides a fertile ground for exploring the connection between spacetime curvature and inertial forces. While general relativity encompasses the dynamics of curved spacetime, its Newtonian limit serves as a benchmark for recovering familiar gravitational phenomena in the weak-field, low-velocity regime.

In this work, we explore the emergence of Newtonian gravity from the proper acceleration field inherent in static de Sitter geometry. Unlike standard derivations that begin with the Einstein field equations, we consider a model where discrete "particles" are defined by localized regions of curvature. This leads directly to the inverse-square gravitational field, offering a fresh geometrical interpretation of mass and force.

Building upon this classical picture, we extend our analysis to the quantum domain. By associating a confining potential with the de Sitter acceleration field, we develop a Schrödinger model for the geometric particle. This quantum description not only complements the classical theory but also provides a platform for exploring gravity as an emergent phenomenon rooted in the quantum structure of spacetime.

2 Classical Derivation from de Sitter Geometry

We begin by considering two copies of flat spacetime, $s_1 = (\mathbb{R}^4, \eta_{\mu\nu})$ and $s_2 = (\mathbb{R}^4, \eta_{\mu\nu})$, and embed spherical regions of s_2 within s_1 as discrete, localized 'geometric particles.' Each such embedded sphere has radius d, and we define $f(r)$ as the number density of these spheres such that the total volume they occupy matches the volume of the outer shell in s_1 of thickness d at radius r:

$$r^4 - (r-d)^4 = f(r)d^4. \tag{1}$$

We interpret each ensemble of $f(r)$ spheres at radius r as constituting a single particle. A total of Z such particles compose the source mass M, resulting in a cumulative embedded volume $Zf(r)d^4$ within radius r.

Next, we analyze the distribution of these spheres centered at positions $r+x$ where $x \in [-d, d]$ and $r \gg d$, forming a quasi-continuous shell. The number of spheres intersecting a thin shell at $r+x$ is $Zdf(r+x) \approx \frac{12Z(r+x)^2}{d^3}dx$, and each intersection contributes a cross-sectional volume $A(x) \approx \frac{4\pi}{3}(d^2-x^2)^{3/2}$. The total cross-sectional volume of a 3-sphere at radius r is $A(r) = 2\pi^2 r^3$.

Within each s_2 sphere, a test mass at a displacement y from its center feels a proper acceleration consistent with the Newtonian limit of static de Sitter space:

$$a(y) = \frac{yc^2}{d^2}. \tag{2}$$

For a sphere centered at $r+x$ from M, the radial component of acceleration is then:

$$a(x) = -\frac{xc^2}{d^2}\hat{r}, \tag{3}$$

where $x/y = -1$ has been used for a uniform distribution across the cross section.

The total average acceleration experienced by a test mass at radius r is computed by integrating over all contributions:

$$a(r) = \int_{-d}^{d} \left(\frac{Zdf(r+x)A(x)}{A(r)} \right) a(x). \tag{4}$$

Now let us define:

$$d = \frac{GM}{Zc^2}, \tag{5}$$

which encapsulates the curvature-to-mass relation. Substituting and simplifying the integral yields:

$$a(r) = \left(-\frac{GM}{r^2} + o\left(\frac{1}{r^2}\right) \right) \hat{r}, \tag{6}$$

as $r \to \infty$.

This shows that the ensemble of embedded de Sitter-like regions leads to an emergent gravitational field that reproduces Newton's law in the weak-field limit.

3 Comparison with Related Work

Prior studies have investigated the Newtonian limit of general relativity and de Sitter spacetime, including the use of Beltrami coordinates and non-inertial frames to model gravitational potentials as limiting behavior. These works often emphasize the asymptotic behavior of spacetime curvature or the transformation properties of geodesics. In contrast, our method starts from the proper acceleration in static de Sitter coordinates and interprets it as a physical force field, then constructs a discrete model from this geometric property. This particle-based approach allows for a direct recovery of Newtonian gravity and a seamless transition to a quantum description.

4 Quantum Description of the Geometric Particle

To extend the classical picture, we consider the "particle" defined as a localized region of curved (de Sitter-like) space, and ask how it might behave under quantum mechanics.

We start with the classical proper acceleration field due to de Sitter geometry:

$$a(r) = \frac{rc^2}{R^2}. \tag{7}$$

This field is the gradient of a potential:

$$V(r) = -\int a(r)\,dr = -\frac{c^2 r^2}{2R^2}. \tag{8}$$

For a stable quantum bound state, we take the opposite sign to obtain a confining potential:

$$V(r) = \frac{1}{2} m\omega^2 r^2, \quad \text{with } \omega = \frac{c}{R}. \tag{9}$$

This is the potential of a 3D isotropic quantum harmonic oscillator.

Radial Schrödinger Equation

In spherical symmetry and for zero angular momentum ($\ell = 0$), the radial Schrödinger equation becomes:

$$-\frac{\hbar^2}{2m}\frac{d^2 u}{dr^2} + \frac{1}{2} m\omega^2 r^2 u(r) = Eu(r), \tag{10}$$

where $\psi(r) = u(r)/r$ is the total wavefunction.

Ground State Solution

The ground state solution is:

$$\psi_0(r) = Ae^{-\frac{m\omega r^2}{2\hbar}}, \tag{11}$$

with energy:

$$E_0 = \frac{3}{2}\hbar\omega = \frac{3}{2}\frac{\hbar c}{R}. \tag{12}$$

The width of the wavefunction is:

$$\Delta r \sim \sqrt{\frac{\hbar}{m\omega}} = \sqrt{\frac{\hbar R}{mc}}. \tag{13}$$

Coupling to Gravity

We may define the mass density as:

$$\rho(r) = m|\psi_0(r)|^2, \tag{14}$$

and use this in Poisson's equation to self-consistently compute a quantum-corrected gravitational field.

This model opens up the possibility of treating gravitational sources as emergent from localized quantum excitations in a curved background geometry.

5 Conclusion

We have shown how Newtonian gravity can be recovered from the mean effect of a discrete particle model in de Sitter geometry, and how such a particle may be treated quantum mechanically as a localized bound state in a harmonic potential. Further work may extend this to full Schrödinger-Poisson systems or semi-classical gravity.